Homework I

1. Show that if a, b and $d>0$ are integers such that $d|a$, $d|b$ and $\left(\frac{a}{d}, \frac{b}{d}\right)=1$, then $(a,b)=d$.

2. Prove that if $a|b$, $b|c$ and $c|d$, then $a|d$.

3. Prove that if $d|a$, $d|b$ and $d|c$ and x, y, z are any integers, then $d|ax+by+cz$.

4. Find all solutions with x and y positive

$$121x + 561y = 13200$$

5. Show that if $a \equiv a' \pmod{m}$ and $b \equiv b' \pmod{m}$, then $a-b \equiv a'-b' \pmod{m}$.

6. Prove that if m and n are positive integers and if $ab \equiv 1 \pmod{m}$ and $a^n \equiv 1 \pmod{m}$, then $b^n \equiv 1 \pmod{m}$.

7. Prove that $7^n \equiv 6n+1 \pmod{36}$ for all positive integers n by induction.

8. Prove by induction that if $a|b_1b_2 \cdots b_nc$ and if $(a, b_i)=1$ for $i=1,\ldots,n$, then $a|c$.